Dynamic stochastic models of general equilibrium in microeconomic modeling

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Outline

1. Equilibrium models
2. Example of model

Equilibrium models

Approaches to macroeconomic modeling

- Stochastic theory is based on the search for economic relations not provided by the theory, as well as testing the reality of the proposed theory.
- Atheoretic modeling – models without the necessary theoretical basis.
- Structural models – models with full theoretical justification. Structural models consist of equations that characterize the basic relationships between economic agents and are based on dynamic models of rational behavior of the economic agent.

Classification of macromodels

- "core" models;
- behavior models focused on future expectations;
- vector autoregression models;
- regression models;
- dynamic optimization models.

Disadvantages of macromodels

- The first macroeconomic models of central banks were based on econometric models of adaptive and rational expectations. The main disadvantage of these models is the disregard for the economic behavior of system agents. These models are not able to adequately predict the consequences of structural changes, the impact of aggregate supply shocks and external factors.

- Since the 1990s, macroeconometric models have given way to computational and stochastic models of general equilibrium. These models already take into account the behavioral strategies of economic entities that influence macroeconomic processes, and describe variable models and databases.

Dynamic stochastic models of general equilibrium

- Dynamic stochastic models of general equilibrium are models of general equilibrium of the economy, which take into account the development and changes of the economic system; the influence of both endogenous and exogenous factors of the environment in which the system operates.

- Dynamic stochastic models of general equilibrium (DSGE-models) is one of the main tools for theoretical study of the impact of monetary and fiscal policy on the economic system.

Principles of DSGE

- The dynamism of the model means that its solution determines the dynamics of all endogenous variables of the system.
- Stochasticity models means that the dynamics of variables are affected by stochastic shocks.
- The general equilibrium means that supply is equal to demand in all markets in all periods.

Model assumptions

▸ economic agents optimize their target functions;

▸ production function and restrictions in the economy are set (technological restrictions, restrictions on monetary and fiscal policy);

▸ uncertainty about the future arises from stochastic shocks that affect the economy and cannot be determined precisely;

▸ expectations in the model are considered rational.

Construction mechanism of DSGE-models

- Formalization of structural equations
- Assessment of long-term equilibrium
- Approximation of the model
- Analysis of properties

Drawback of DSGE-models

▸ Large size of models, which is effective in a process modeling economic speakers in long-term perspective, is one of basic shortcomings application data models.

▸ The complexity of the presentation requires a significant mathematical apparatus.

DSGE models

- DSGE models based on utilities, technologies and others features of economy
- DSGE models capable to explain dynamics variables in in terms theory
- Forecasts quality of DSGE models is almost equal to VAR models
- DSGE models CB: SIGMA (US Federal Reserve), NAWM (ECB), BEQM (Bank of England), ToTEM (Bank of Canada), GEM (IMF), NEMO (Norway), RAMSES (Sweden), MAS (Chile), MEGA-D (Peru)

Example of model

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The structure is simple DSGE models
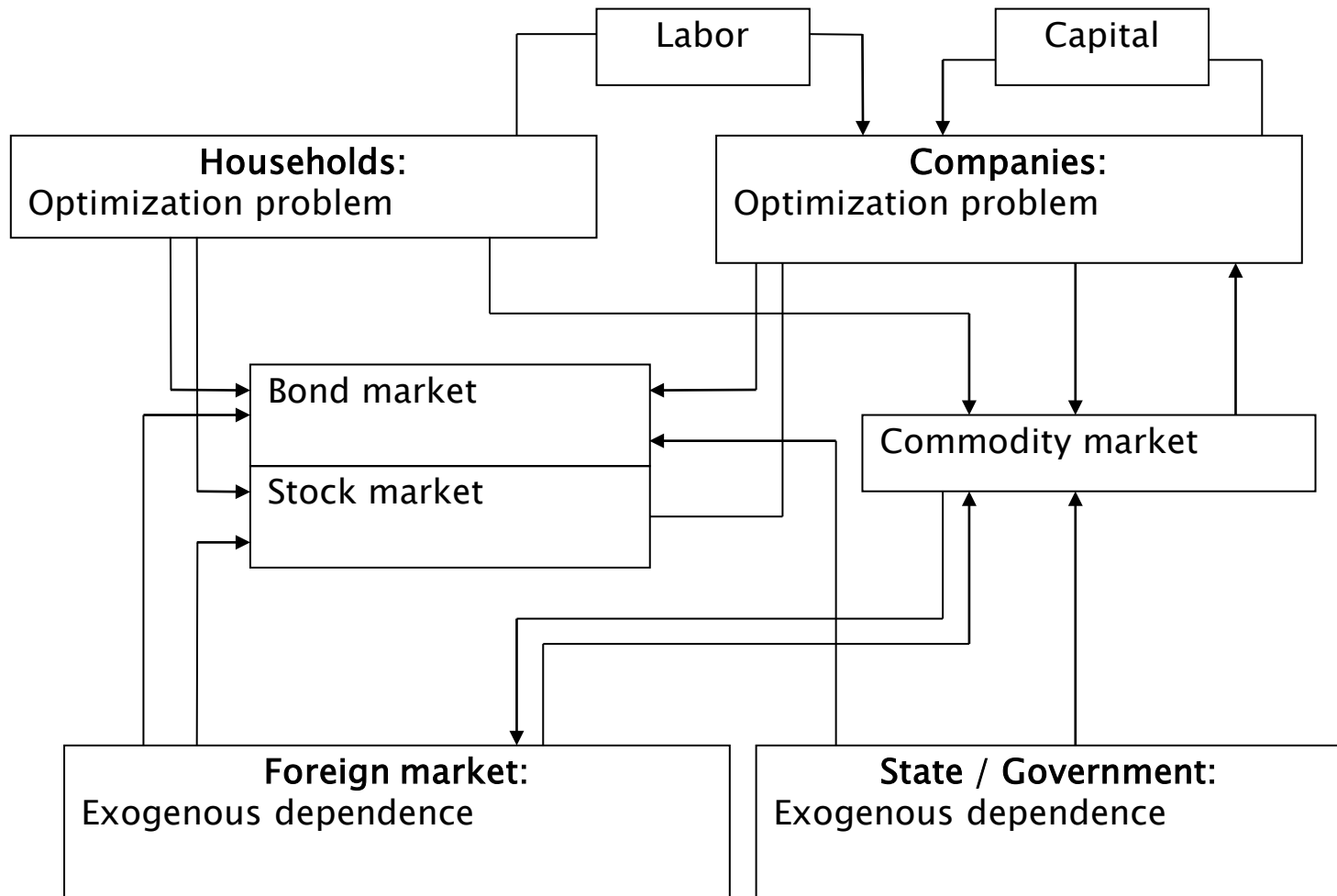
Households: preferences

- Like consume goods,
- Dislike to work,
- Like to have money
- Dislike to have stocks and bonds.

Households: budget constraint

- postpon money,
- spend money on goods,
- spend money on domestic and foreign bonds,
- spend money domestic and foreign shares,
- receive money, postponed in the past period,
- receive money from repayment domestic and foreign bonds,
- receive money from sales of domestic and foreign shares,
- receive dividends on shares and bonds,
- receive salary,
- to pay income taxes.

Firms: preferences

- Want pay more dividends,
- Dislike to change output, prices or number of employees.

Firms: budget constraint

- pay dividends,
- buy investment goods,
- pay salary to employees,
- receive revenues for produced goods,
- pay sales tax.

State: rules

- sells so many bonds that interest rates remain at a given level (depending on inflation, economic growth, public debt and previous rates).

State: budget constraint

- spends money on purchase bonds,
- spends money on purchase goods and labor,
- receives funds from repayment bonds previous period,
- receives funds from taxes on income from labor and release,
- receives funds from printing money and sales bonds.

External market: rules

▸ the number of domestic shares purchased abroad is described by a random process.

Foreign market: budget constraint

- spends funds for purchase domestic and foreign shares and bonds,
- spends funds for payment export,
- receives money from imports,
- receives money from repayment domestic and foreign bonds,
- receives money from sales shares and dividends from shares.

Parameters: B

- $B_{F, D, t}$ – amount of bonds in domestic currency purchased foreign market in period t
- $B_{F, F, t}$ – amount of bonds in foreign currency purchased foreign market in period t
- $B_{G, D, t}$ – amount of bonds in domestic currency purchased state in period t
- $B_{H, D, t}$ – amount of bonds in national currency purchased households in period t
- $B_{H, F, t}$ – amount of bonds in foreign currency purchased households in period t

Parameters : C

- $C_{D,t}$ – the volume of consumption of domestic goods in the period t
- C_t – volume of consumption in the period t

Parameters : D

- $D_{D,\,t}$ – dividends on domestic shares in the period t
- $D_{F,\,t}$ – dividends on foreign shares in the period t

Parameters : F

- F_t – exchange rate in the period t (number of units of national currency in exchange for a unit of foreign currency)

Parameters: G

- $G_{D,\,t}$ – government spending on domestic goods in the period t
- G_t – government spending in the period t

Parameters: *I*

- $I_{D,\,t}$ – demand for domestic investment in the period t
- I_t – demand for investment in the period t

Parameters: K

- K_t – the amount of capital in the period t

Parameters: L

- L_t – labor supply in the period t
- $L_{f,\,t}$ – demand for labor from firms in the period t
- $L_{G,\,t}$ – demand for labor from the state in the period t

Parameters: M

- M_t – money supply in the period t

Parameters: P

- $P_{D,\,t}$ – the level of prices for domestic goods in the period t
- $P_{F,\,t}$ – the level of prices for foreign goods in the period t
- P_t – price level in the period t

Parameters: R

- $R_{D,\,t}$ – interest rate on bonds in national currency in the period t
- $R_{F,\,t}$ – interest rate on foreign currency bonds in the period t

Parameters: S

- $S_{D,\,t}$ – the price of domestic shares in the period t
- $S_{F,\,t}$ – the price of foreign shares in the period t

Parameters: T

- $T_{L,t}$ – labor tax rate in the period t
- $T_{Y,t}$ – sales tax rate in the period t

Parameters: W

- W_t – the level of wages in the period t

Parameters: X

- $X_{F, D, t}$ – the number of shares of domestic companies purchased abroad in the period t
- $X_{F, F, t}$ – the number of shares of foreign companies purchased abroad in the period t
- $X_{H, D, t}$ – the number of shares of national companies purchased by households in the period t
- $X_{H, F, t}$ – the number of shares of foreign companies purchased by households in the period t

Parameters: Y

- $Y_{D,\,t}$ – issue of domestic firms in the period t
- $Y_{E,\,t}$ – exports in the period t
- $Y_{F,\,t}$ – foreign demand for domestic goods in the period t
- $Y_{I,\,t}$ – imports in the period t
- Y_t – domestic production in the period t

Parameters: Z

- $Z_{\beta, t}$ – Exogenous process, modelling interperiod household preferences
- $Z_{F, B, t}$ – Exogenous process that characterizes the number of bonds in domestic currency purchased abroad
- $Z_{F, D, t}$ – Exogenous process that characterizes dividends on foreign stocks
- $Z_{F, P, t}$ – Exogenous process that characterizes the dynamics of prices abroad
- $Z_{M, t}$ – Exogenous process that characterizes the dynamics of money supply abroad
- $Z_{L, t}$ – Exogenous process that characterizes the dynamics of demand for work abroad
- $Z_{Y, t}$ – Exogenous process that characterizes the dynamics of demand for goods abroad

Households: target function

$$E_0 \sum_{t=0}^{\infty} \beta^t Z_{\beta,t} \left(\begin{array}{l} \dfrac{(C_t)^{1+\omega_C}}{(Z_{Y,t} Z_{P,t})^{1+\omega_C}(1+\omega_C)} - Z_{L,t} \dfrac{(L_t / Z_{P,t})^{1+\omega_L}}{1+\omega_L} + \\[3mm] + Z_{M,t} \left(\dfrac{M_t}{P_t(Z_{Y,t} Z_{P,t})} \right)^{\omega_M+1} \dfrac{1}{1+\omega_M} - \dfrac{\varphi_{X,F}}{2} \left(X_{H,F,t} - \overline{X}_{H,F} \right)^2 \\[3mm] - \dfrac{\varphi_{X,D}}{2} \left(X_{H,D,t} - \overline{X}_{H,D} \right)^2 - \dfrac{\varphi_{B,F}}{2} \left(B_{H,F,t} - \overline{B}_{H,F} \right)^2 - \\[3mm] - \dfrac{\varphi_{B,D}}{2} \left(B_{H,D,t} - \overline{B}_{H,D} \right)^2 \end{array} \right) \to \max_{C;M;B;L;X}$$

Households: budget constraint

$$P_t C_t + \begin{pmatrix} B_{H,D,t} + M_t + X_{H,D,t} S_{D,t} + \\ + F_t B_{H,F,t} + X_{H,F,t} F_t S_{F,t} \end{pmatrix} = W_t L_t + \begin{pmatrix} R_{D,t-1} B_{H,D,t-1} + M_{t-1} + \\ + X_{H,D,t-1}\left(S_{D,t} + D_{D,t}\right) + \\ + F_t R_{F,t-1} B_{H,F,t-1} + \\ + X_{H,F,t-1} F_t \left(S_{F,t} + D_{F,t}\right) \end{pmatrix}$$

Firms: target function

$$E_0 \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} R_{D,s} \right)^{-1} D_t \left(\begin{array}{l} 1 + \varphi_P \left(\ln\left(\dfrac{P_t}{P_{t-1}}\right) - \overline{P} \right)^2 + \varphi_L \left(\ln\left(\dfrac{L_t}{L_{t-1}}\right) - \overline{L} \right)^2 + \\[2em] + \varphi_Y \left(\ln\left(\dfrac{Y_t}{Y_{t-1}}\right) - \overline{Y} \right)^2 \end{array} \right) \rightarrow \max_{D;P;Y;L;I;K}$$

Firms: restrictions

- Budget constraint:

$$W_t L_t + P_t I_t + D_t = \left(1 - \tau_{Y,t}\right) P_t Y_t$$

- Demand restrictions:

$$Y_t = \left(\frac{P_t}{P_{D,t}}\right)^{-\theta} \left(C_{D,t} + I_{D,t} + G_{D,t}\right) + \left(\frac{P_t}{P_{F,t}}\right)^{-\theta} \left(Y_{F,t}\right)$$

- Production function:

$$Y_t = \left(Z_{Y,t} L_t\right)^{\alpha} K_{t-1}^{1-\alpha}$$

- Evolution of capital:

$$K_t = \left(1 - \delta\right) K_{t-1} + I_t$$

State: budget constraint

$$P_t G_t + W_t L_{G,t} + M_{t-1} + B_{G,t} = M_t + R_{D,t-1} B_{G,t-1} + \tau_{Y,t} P_{D,t} Y_{D,t} + \tau_{L,t} W_t L_t$$

Foreign market: budget constraints

$$\begin{pmatrix} X_{F,D,t}S_{D,t} + X_{F,F,t}F_tS_{F,t} + \\ + F_tB_{F,F,t} + B_{F,D,t} + P_{D,t}Y_{E,t} \end{pmatrix} = \begin{pmatrix} X_{F,D,t-1}(S_{D,t} + D_{D,t}) + X_{F,F,t-1}F_t(S_{F,t} + D_{F,t}) + \\ + F_tR_{F,t-1}B_{F,F,t-1} + R_{D,t-1}B_{F,D,t-1} + F_tP_{F,t}Y_{I,t} \end{pmatrix}$$

$$\mathrm{R}_{F,t} = Z_{F,R,t}$$

$$\frac{(S_{F,t} + D_{F,t})}{S_{F,t-1}} = e^{r_{F,t} + z_{F,S,t}}$$

$$\frac{(D_{F,t})}{P_{F,t}Y_{F,t}} = e^{z_{F,D,t}}$$

$$X_{F,D,t} = Z_{F,X,t}$$

$$\frac{F_tP_{F,t}}{P_t} = Z_{F,P,t}$$

$$\frac{Y_{F,t}}{Z_{P,t}Z_{Y,t}} = Z_{F,Y,t}$$

$$\frac{B_{F,D,t}}{P_tZ_{P,t}Z_{Y,t}} = Z_{F,B,t}$$

Foreign market: balance restrictions – 1

$$B_{H,D,t} + B_{F,D,t} + B_{G,D,t} = 0$$

$$B_{H,F,t} + B_{F,F,t} = 0$$

$$X_{H,F,t} + X_{F,F,t} = 1$$

$$X_{H,D,t} + X_{F,D,t} = 1$$

$$L_t = L_{f,t} + L_{G,t}$$

$$C_{D,t} = \left(\frac{P_{D;t}}{P_t} \right)^{-\theta} w_D C_t$$

$$I_{D,t} = \left(\frac{P_{D;t}}{P_t} \right)^{-\theta} w_D I_t$$

$$G_{D,t} = \left(\frac{P_{D;t}}{P_t} \right)^{-\theta} w_D G_t$$

Foreign market: balance restrictions – 2

$$P_t = \left(\omega_D P_{D,t}^{1-\theta} + (1-\omega_D) F_t^{1-\theta} P_{F,t}^{1-\theta}\right)^{1/(1-\theta)}$$

$$Y_{I,t} = \left(\frac{F_t P_{F;t}}{P_t}\right)^{-\theta} (1-\omega_D)(C_t + I_t + G_t)$$

$$Y_{E,t} = \left(\frac{P_{D;t}}{F_t P_{F;t}}\right)^{-\theta} (Y_{F,t})$$

Stationarity of processes

- Typically, all variables are converted to stationary using:
  - normalization;
  - price indexation;
  - logarithm;
  - other approaches.

Application of the model

- Forecasting of model variables
- Explanation of events
- Defining state policy to achieve certain goals

Modeling

▸ For evaluation DSGE models developed a special program Dinare

Thank you!